A Very Rough Introduction to Linear Logic

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Multicore Group Seminar

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Linear Logic

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- Classical logic: is my formula *true*?
- Intuitionistic logic: is my formula *provable*?
- Linear logic is a bit different

Vending machines

Let:

$$\begin{aligned} D &:= \text{“One dollar”} \\ M &:= \text{“A pack of Marlboros”} \\ C &:= \text{“A pack of Camels”} \end{aligned}$$

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$$D \Rightarrow M$$

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$$D \Rightarrow M$$
$$D \Rightarrow C$$

In classical or intuitionistic logic, we can deduce:

$$D \Rightarrow (M \wedge C)$$

Linear logic operators

Two conjunctions:

- **Multiplicative conjunction** ($A \otimes B$):
  I can have both A and B at the same time
- **Additive conjunction** ($A \,\&\, B$):
  I can have A or B (not both), and I choose which

Linear logic operators

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Two disjunctions:

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  [we'll come back to this one]
- **Additive disjunction** ($A \oplus B$):
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De Morgan rules:

$$
\begin{aligned}
(A \otimes B)^\perp &\equiv A^\perp \parr B^\perp \\
(A \mathbin{\&} B)^\perp &\equiv A^\perp \oplus B^\perp
\end{aligned}
$$

More vending machines

Let R be a vending machine that accepts rubles and dispenses packs of rolos, and let S be a vending machine that accepts shillings and dispenses packs of softmints.

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- $P_4 \multimap (R \parr S)$: If Ally hands in purchase order P_4, the college will install both machines side-by-side, and pre-load one of them with a coin. If a customer puts a coin in the other slot, then *one of the machines* will consume its coin and dispense its confectionery!

Exponentials

- $!A$: Produces an unlimited number of A's
- $?A$: Consumes an unlimited number of A's
- Connected via linear negation: $(?A)^\perp = !(A^\perp)$

Jeroen's Burger Shack

Set menu – $5 per person
Hamburger
Fries or Wedges
Unlimited Pepsi
Ice-cream or sorbet (subject to availability)

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$$D \otimes D \otimes D \otimes D \otimes D$$
$$\multimap$$
$$H \otimes (F \mathbin{\&} W) \otimes {!P} \otimes (I \oplus S)$$

Gentzen's LK

The sequent

$$A_1, \ldots, A_m \vdash B_1, \ldots, B_n$$

means

$$(A_1 \wedge \cdots \wedge A_m) \Rightarrow (B_1 \vee \cdots \vee B_n)$$

Gentzen's LK vs. Linear Logic

$$\frac{}{A \vdash A}\ \textsc{Axiom} \qquad \frac{\Gamma,\ A,\ A \vdash \Delta}{\Gamma,\ A \vdash \Delta}\ \textsc{Cont}_L \qquad \frac{\Gamma \vdash\ A,\ A, \Delta}{\Gamma \vdash\ A, \Delta}\ \textsc{Cont}_R$$

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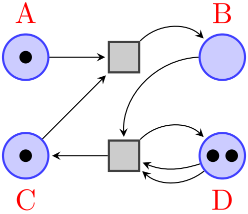
Application: Petri nets



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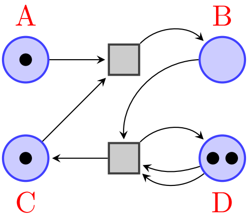

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$$A \otimes C \otimes D \otimes D$$

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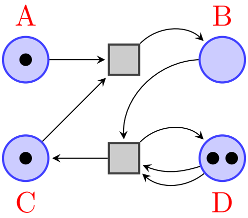

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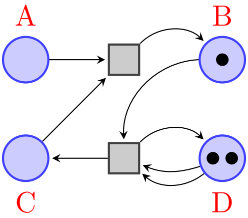

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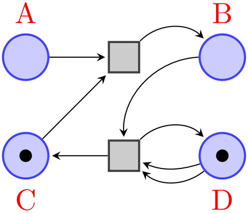

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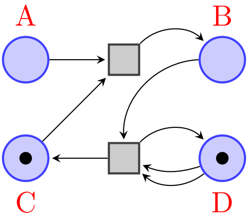

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- Other applications include: finding optimisations in Haskell, logic programming, quantum physics, linguistics, . . .

Connection to separation logic

- Linear logic models the *number of times* a resource is used.
- To reason about computer memory, we want to control how resources are *shared*.
- The logic of *bunched implications* (BI) does this. (Separation logic is based on BI.)

The idea of BI

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- Sequents in BI have this form:

$$\mathcal{B} \vdash A$$

where $\mathcal{B} ::= A \mid \mathcal{B}, \mathcal{B} \mid \mathcal{B}; \mathcal{B}$

Some proof rules for BI

- Weakening and Contraction are allowed for ';' but not for ','

$$\frac{\mathcal{B} \vdash A}{\mathcal{B}; \mathcal{B}' \vdash A}\ \text{W} \qquad\qquad \frac{\mathcal{B}; \mathcal{B} \vdash A}{\mathcal{B} \vdash A}\ \textsc{Cont}$$

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- The ';' corresponds to '∧' (ordinary conjunction)

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- The ',' corresponds to '$*$' (separating conjunction)

$$\frac{\mathcal{B} \vdash A \qquad \mathcal{B}' \vdash A'}{\mathcal{B}, \mathcal{B}' \vdash A * A'}\ *\mathrm{I}$$

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Linear logic . . .

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- models production and consumption of resources (but not sharing)
- has two types of conjunction, and two types of disjunction
- has exponentials, which relax the strict rules on resource usage
- is obtained by removing WEAKENING and CONTRACTION from classical logic
- has various applications, e.g. modelling Petri nets

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