More Separation Logic

Lecturer: John Wickerson

Lecture Plan

- A old-style proof of `list_reverse`
- A proof of `list_reverse` in separation logic
- Separation logic's proof rules
- Soundness of the Frame rule

References

- Mike Gordon. *Hoare Logic.* Lecture Notes, 2011.
- John Reynolds. *Separation Logic: A Logic for Shared Mutable Data Structures.* LICS 2002.
- Peter O'Hearn, John Reynolds and Hongseok Yang. *Local Reasoning about Programs that Alter Data Structures.* CSL 2001.

Lecture Plan

- A old-style proof of `list_reverse`
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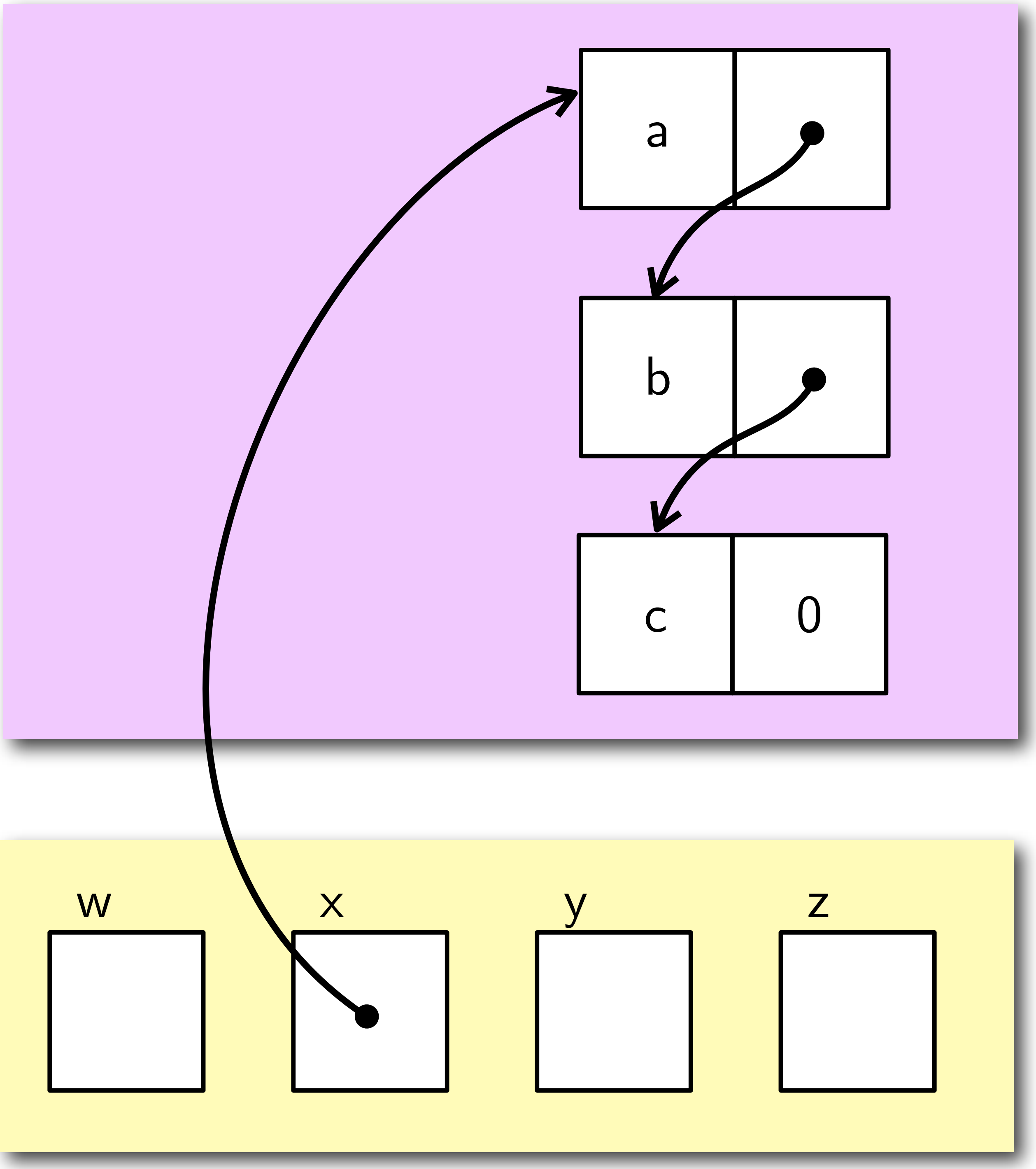
Proof of list reverse

{list δ x}

```
y := 0;
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
```

{list -δ y}



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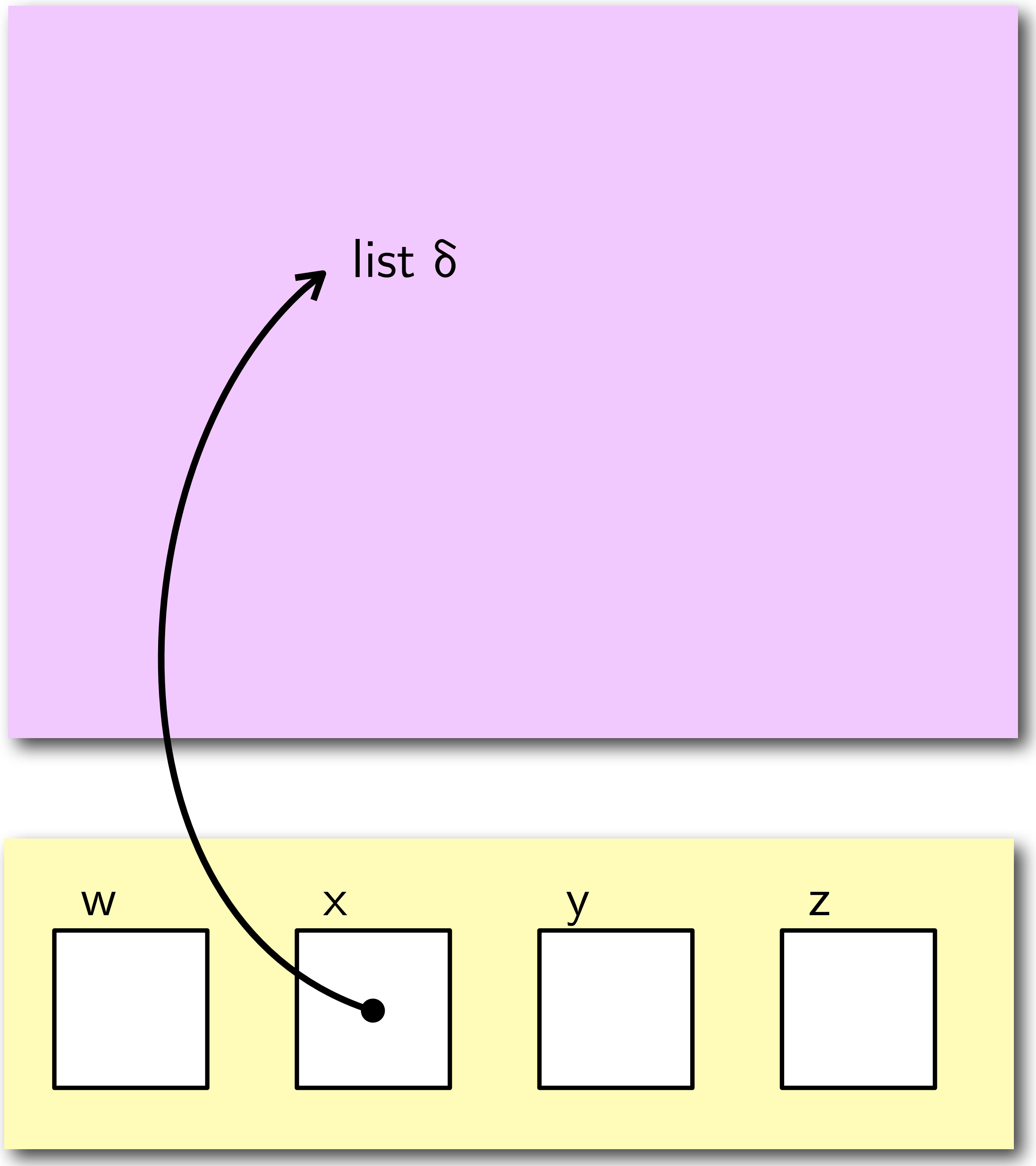

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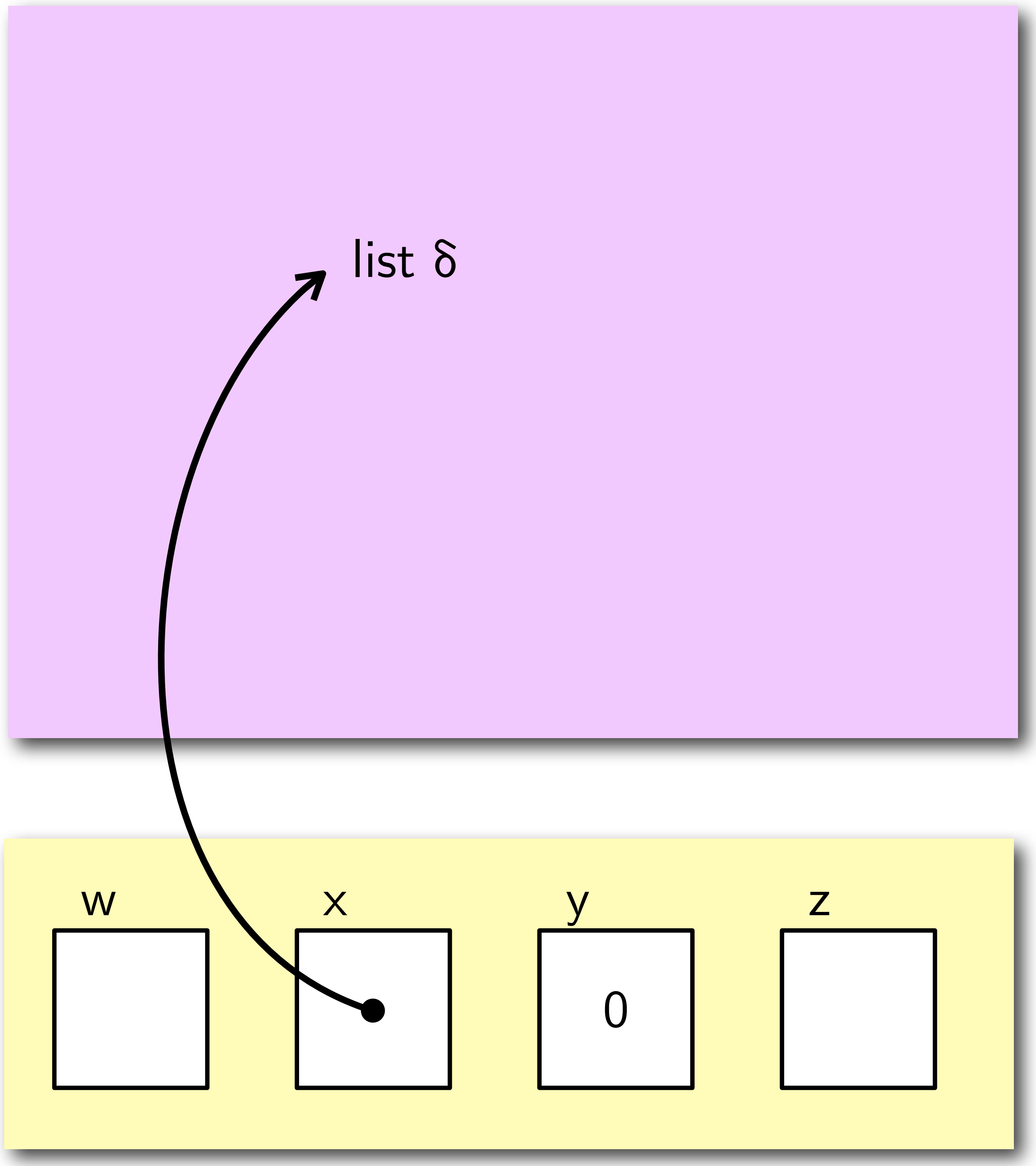


Proof of list reverse

{list δ x}

y := 0;

{∃α,β. list α x ∧ list β y ∧ δ = -β·α}

while (x≠0) do {

    z := [x+1];

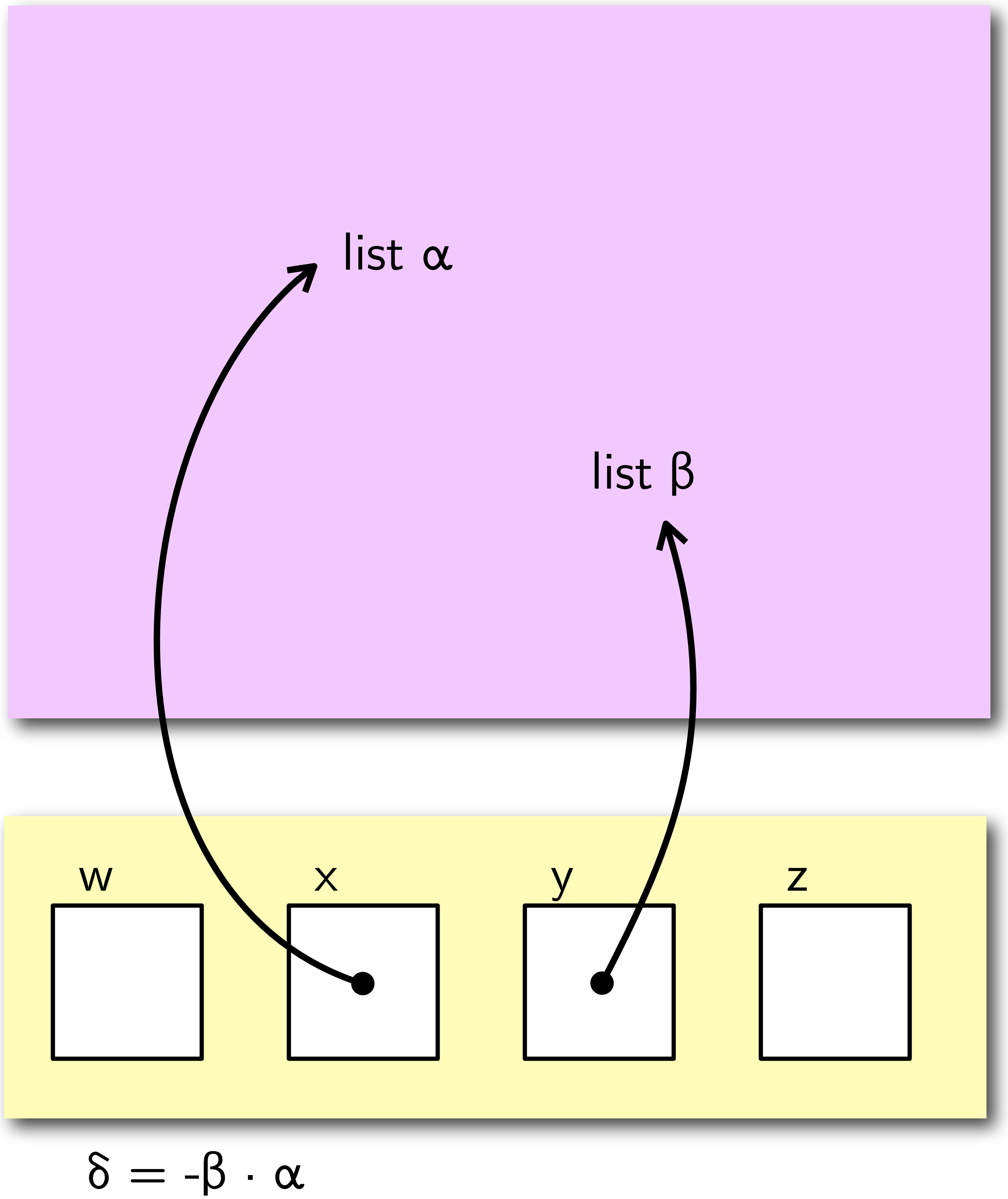
    [x+1] := y;

    y := x;

    x := z;

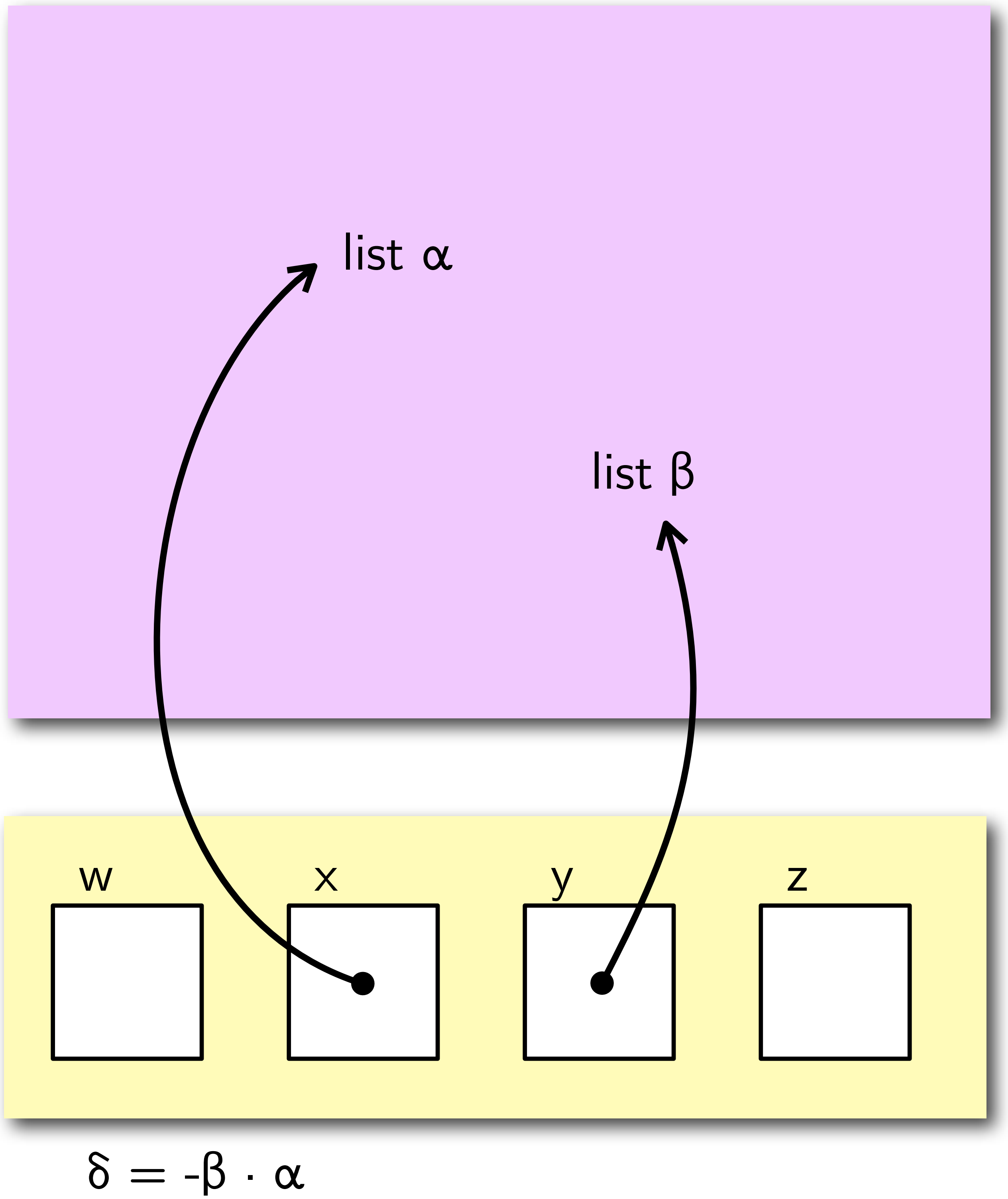
}

{list -δ y}



Proof of list reverse

{list δ x}
y := 0;
{∃α,β. list α x ∧ list β y ∧ δ = -β·α
∧ (∀z. reach(x,z) ∧ reach(y,z) ⇒ z=0)}
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
{list -δ y}



Proof of list reverse

$$\mathrm{reach}(x,y) \;=\; \exists n \geq 0.\ \mathrm{reach}_n(x,y)$$

$$\mathrm{reach}_0(x,y) \;=\; x=y$$

$$\mathrm{reach}_{n+1}(x,y) \;=\; \exists x'.\ x \hookrightarrow \_,x' \wedge \mathrm{reach}_n(x',y)$$

{list δ

y :=

{∃α,β

∧ (∀z

while

z

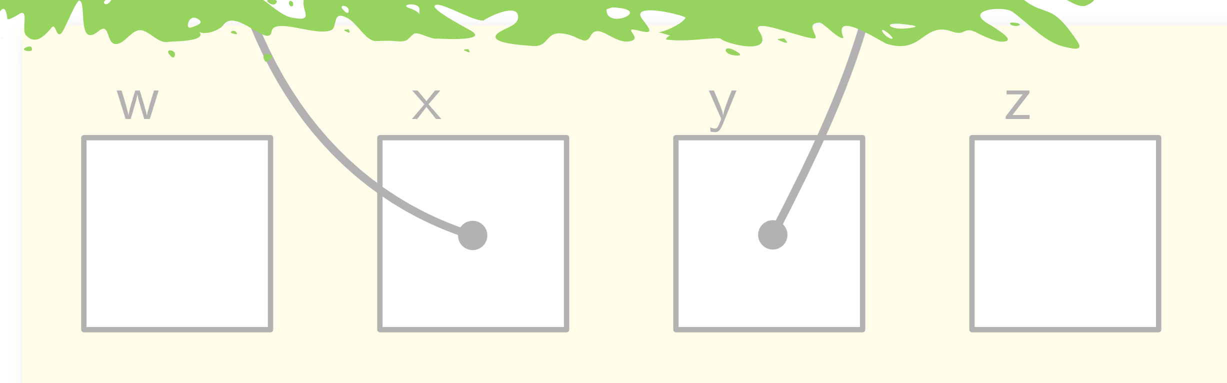
[x+1] := y;

y := x;

x := z;

}

{list -δ y}



δ = -β · α

Proof of list reverse

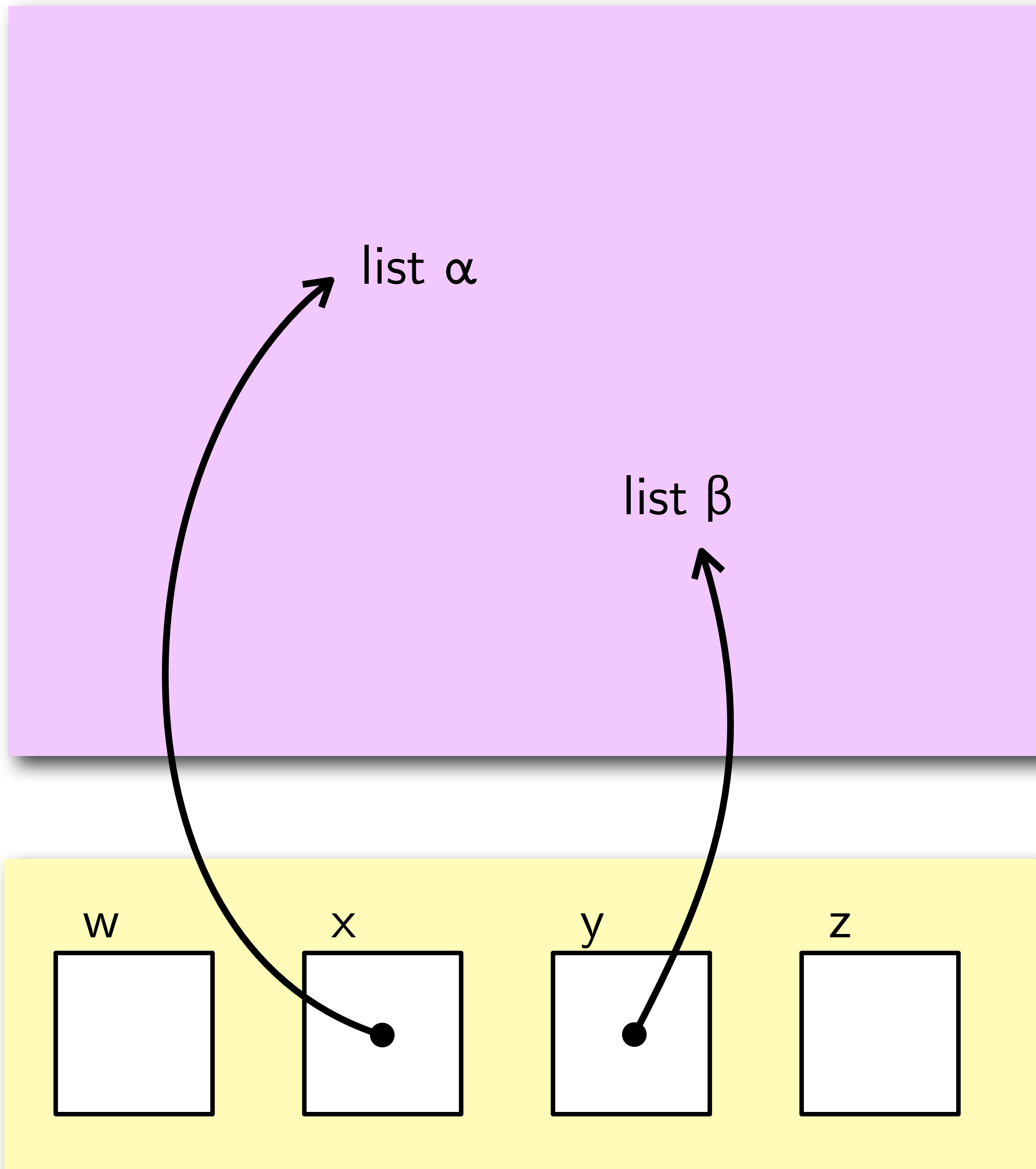
{list δ x}

y := 0;

{∃α,β. list α x ∧ list β y ∧ δ = -β·α
∧ (∀z. reach(x,z) ∧ reach(y,z) ⇒ z=0)}

```
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
```

{list -δ y}



δ = -β · α

Proof of list reverse

{list δ x}
list_reverse(x,y)
{list -δ y}

Proof of list reverse

{list δ x ∧ list ε w}
list_reverse(x,y)
{list -δ y}

Proof of list reverse

{list δ x $\wedge$ list ε w
$\wedge$ ($\forall$z. reach(x,z) $\wedge$ reach(w,z) $\Rightarrow$ z=0)}
list_reverse(x,y)
{list -δ y}

Proof of list reverse

{list δ x ∧ list ε w
∧ (∀z. reach(x,z) ∧ reach(w,z) ⇒ z=0)}

```
y := 0;
```

{∃α,β. list α x ∧ list β y ∧ δ = -β·α
∧ (∀z. reach(x,z) ∧ reach(y,z) ⇒ z=0)}

```
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
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{list -δ y}

Proof of list reverse

{list δ x ∧ list ε w
∧ (∀z. reach(x,z) ∧ reach(w,z) ⇒ z=0)}

y := 0;

{∃α,β. list α x ∧ list β y ∧ δ = -β·α
∧ (∀z. reach(x,z) ∧ reach(y,z) ⇒ z=0)

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∧ (∀z. (reach(x,z) ∨ reach(y,z))
∧ reach(w,z) ⇒ z=0)}

```
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
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```

{list -δ y}

Proof of list reverse

{list δ x ∧ list ε w
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y := 0;
```

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```
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
```

{list -δ y ∧ list ε w
∧ (∀z. reach(x,z) ∧ reach(w,z) ⇒ z=0)}

Proof of list reverse

{list δ x ∧ list ε w
∧ (∀z. reach(x,z) ∧ reach(w,z) ⇒ z=0)}

list_reverse(x,y)

{list -δ y ∧ list ε w
∧ (∀z. reach(x,z) ∧ reach(w,z) ⇒ z=0)}}

20th century proof

Summary:

Without separation logic, proofs are **fiddly** and **not modular**, but they **can be done**.

Lecture Plan

- A old-style proof of `list_reverse`
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Proof of list reverse

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y := 0;
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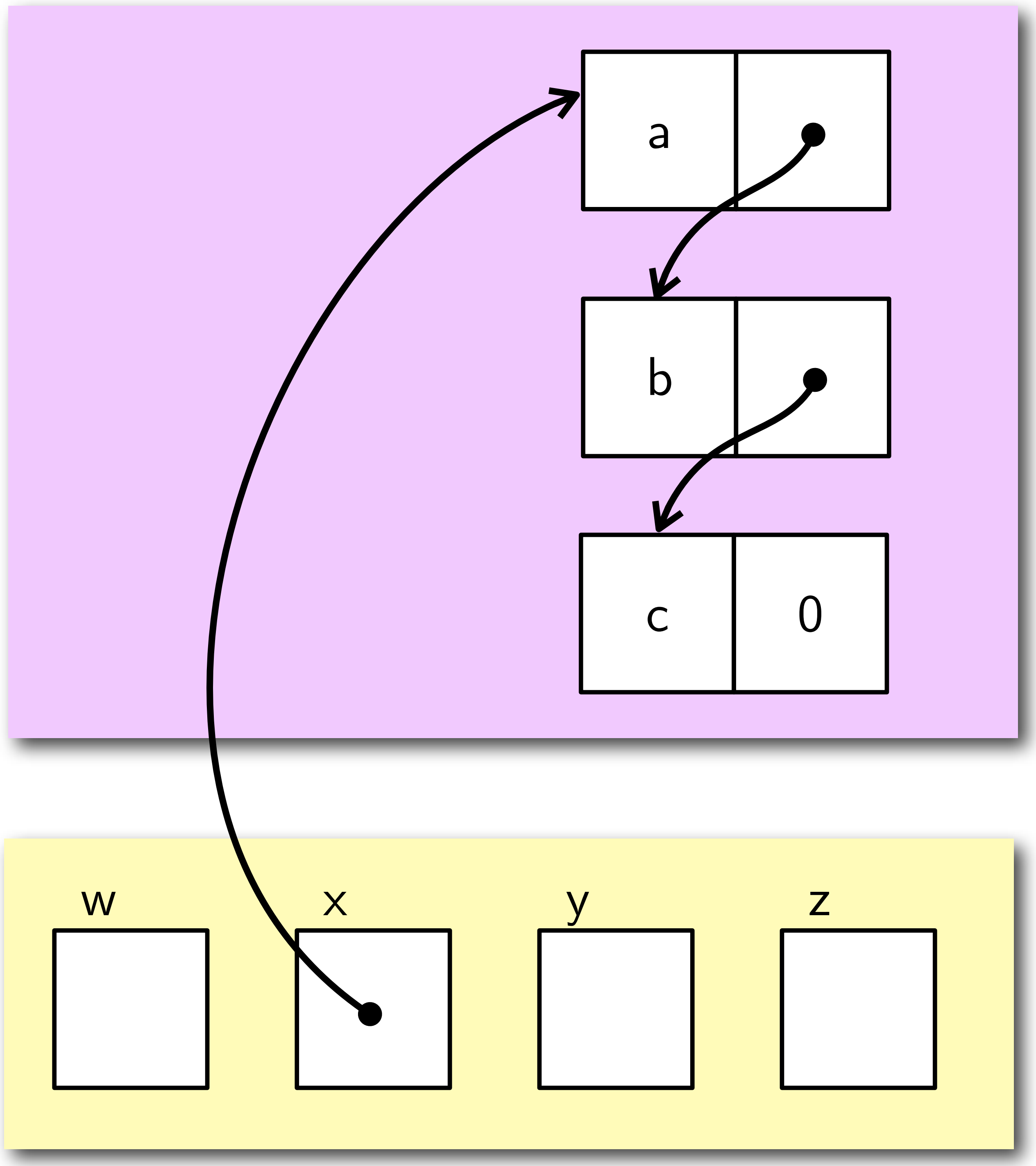
{list -δ y}



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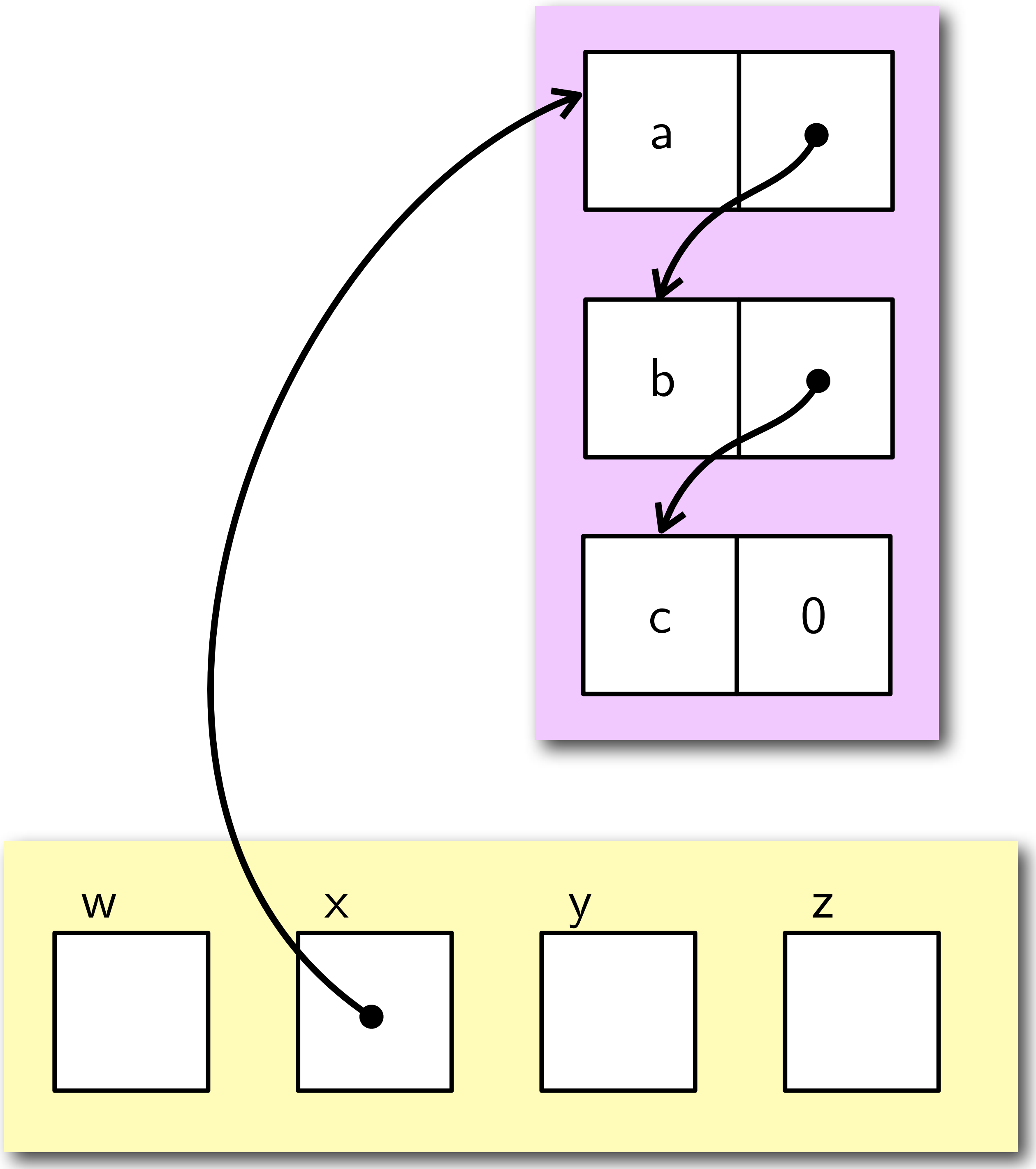

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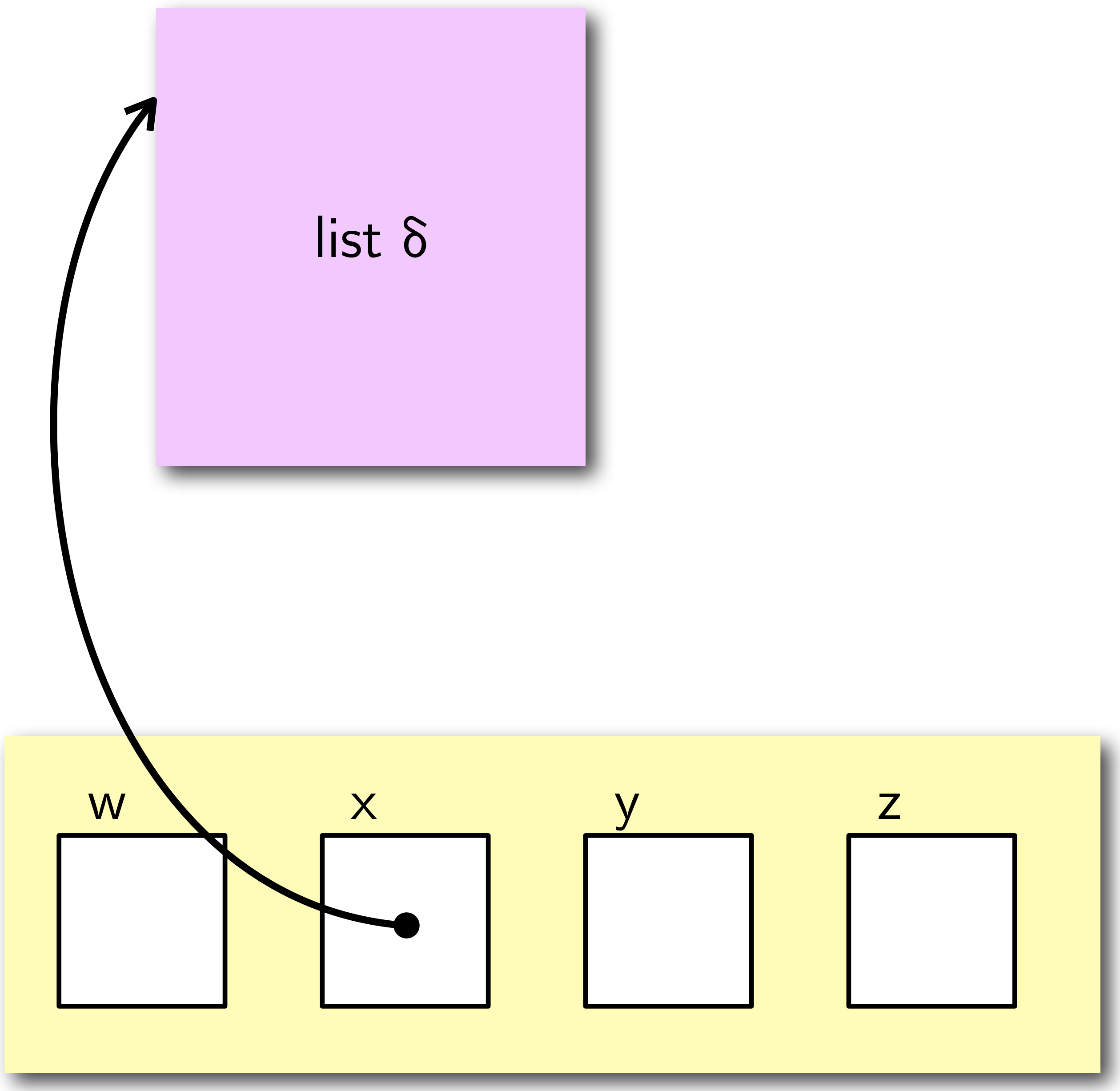

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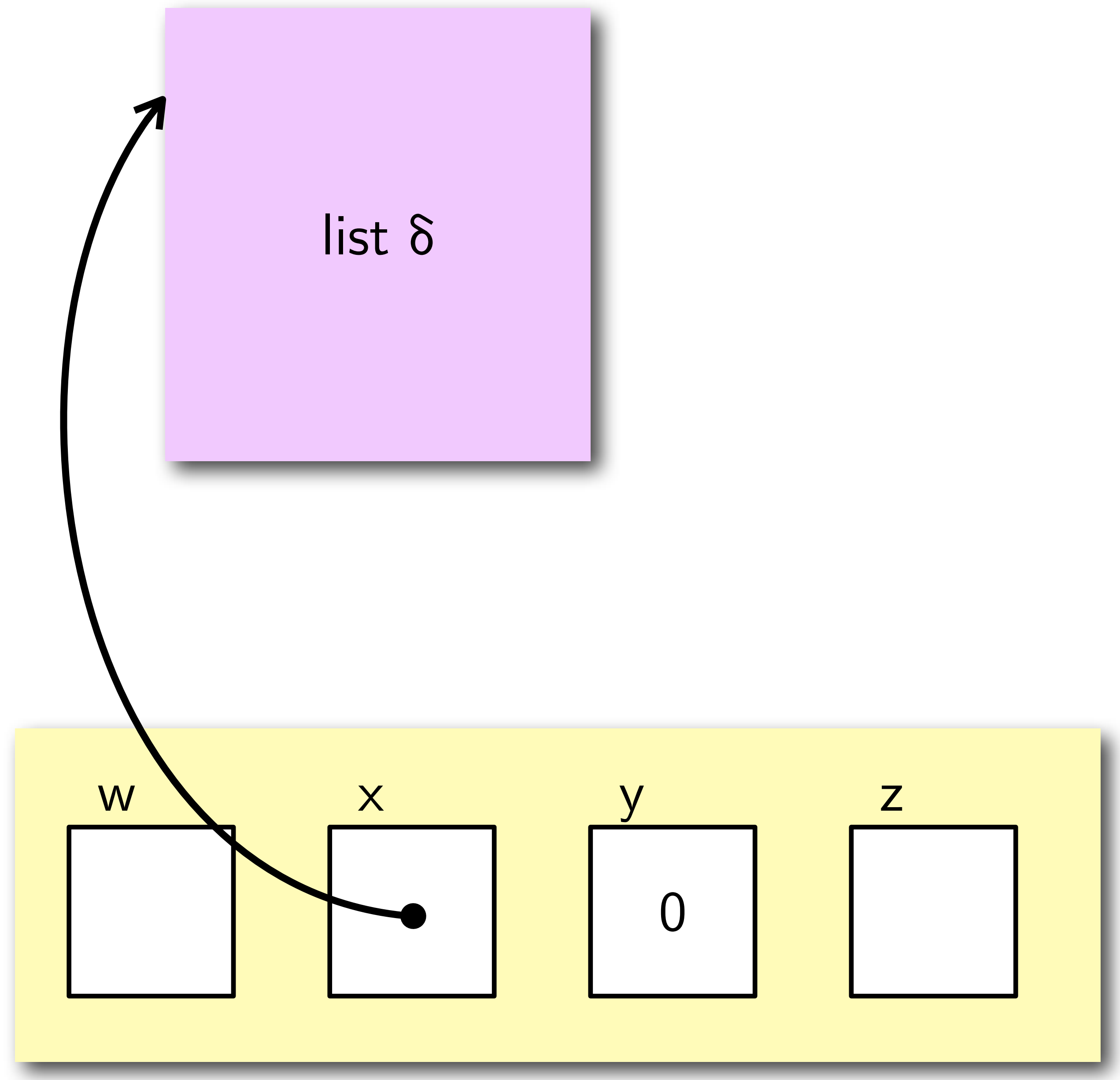


Proof of list reverse

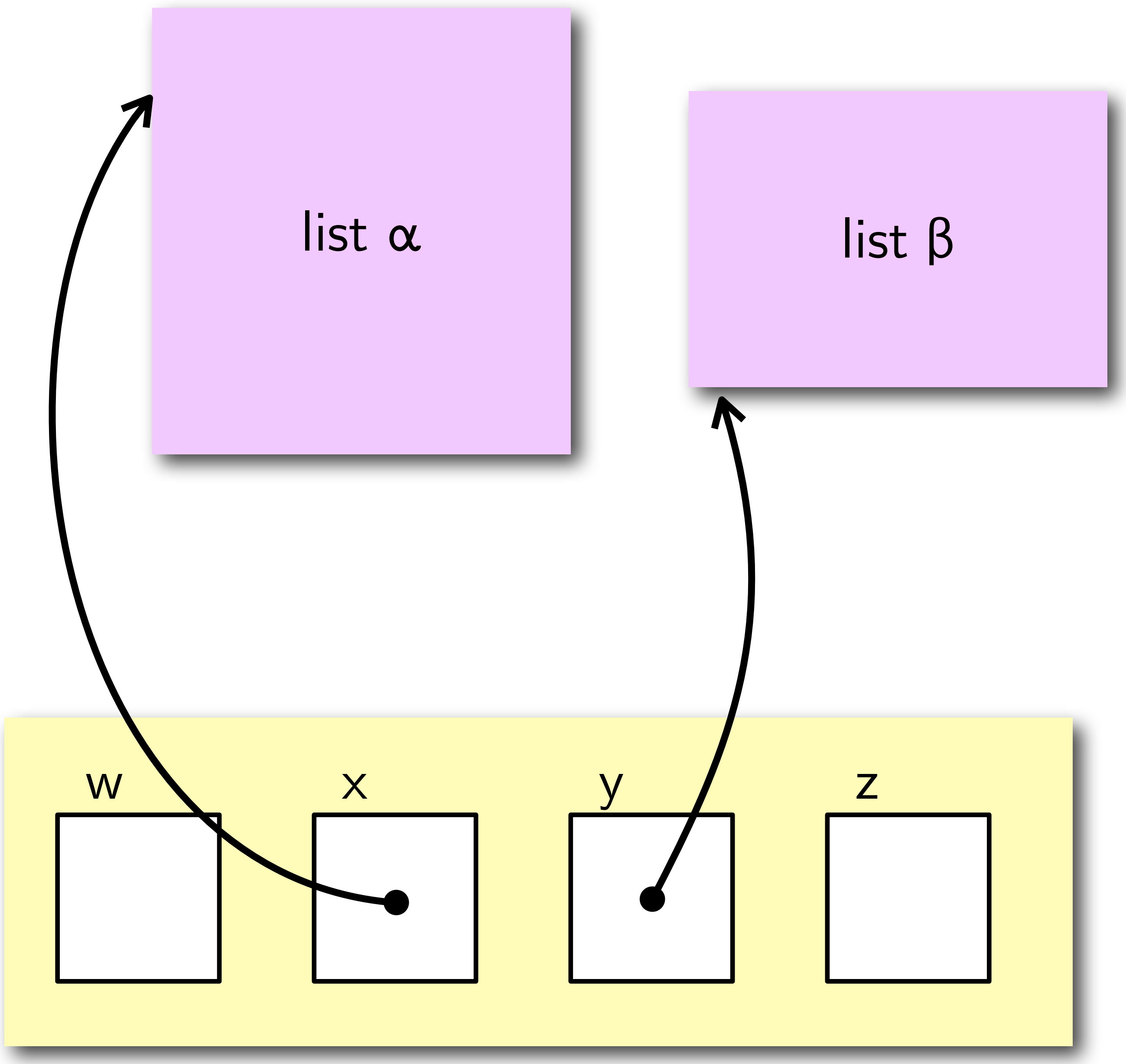
{list δ x}

y := 0;

{∃α,β. list α x ∗ list β y ∗ δ ≐ -β·α}

```
while (x≠0) do {
    z := [x+1];
    [x+1] := y;
    y := x;
    x := z;
}
```

{list -δ y}



δ = -β · α

Proof of list reverse

$\{\text{list } \delta \ x\}$

y := 0;

while $\{\exists \alpha,\beta.\ \text{list } \alpha \ x * \text{list } \beta \ y * \delta \doteq {-\beta}\cdot\alpha\}$ (x≠0) do {

  $\{\exists a,\alpha,\beta,Z.\ x \mapsto a,Z * \text{list } \alpha \ Z * \text{list } \beta \ y * \delta \doteq {-\beta}\cdot a\cdot\alpha\}$

  z := [x+1];

  $\{\exists a,\alpha,\beta.\ x \mapsto a,z * \text{list } \alpha \ z * \text{list } \beta \ y * \delta \doteq {-\beta}\cdot a\cdot\alpha\}$

  [x+1] := y;

  $\{\exists a,\alpha,\beta.\ x \mapsto a,y * \text{list } \alpha \ z * \text{list } \beta \ y * \delta \doteq {-\beta}\cdot a\cdot\alpha\}$

  $\{\exists \alpha,\beta.\ \text{list } \alpha \ z * \text{list } \beta \ x * \delta \doteq {-\beta}\cdot\alpha\}$

  y := x; x := z;

  $\{\exists \alpha,\beta.\ \text{list } \alpha \ x * \text{list } \beta \ y * \delta \doteq {-\beta}\cdot\alpha\}$

}

$\{\text{list } {-\delta} \ y\}$

Proof of list reverse

{list δ x}
list_reverse(x,y)
{list -δ y}

Proof of list reverse

{list δ x ∗ list ε w ∗ tree t}
list_reverse(x,y)
{list -δ y}

Proof of list reverse

{list δ x ∗ list ε w ∗ tree t}
list_reverse(x,y)
{list -δ y ∗ list ε w ∗ tree t}

$$\frac{\{P\}\ C\ \{Q\}}{\{P * R\}\ C\ \{Q * R\}}(\dagger)$$

†provided R doesn't mention any variable modified by C

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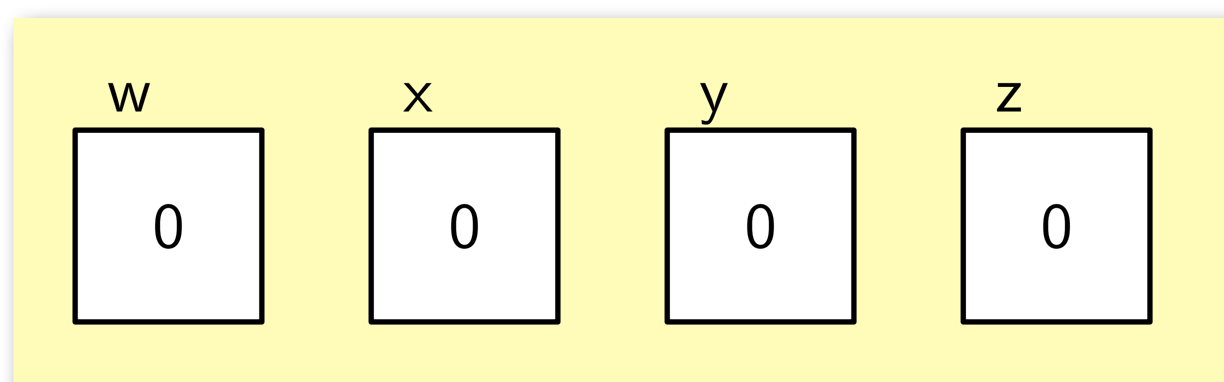
```
x := cons(3);
y := cons(5,1);
x := [x];
[y+1] := 6;
dispose(y);
```



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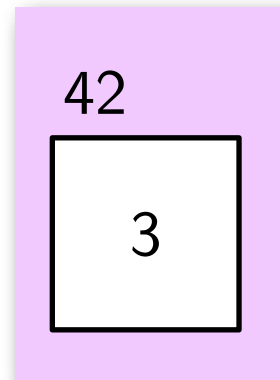

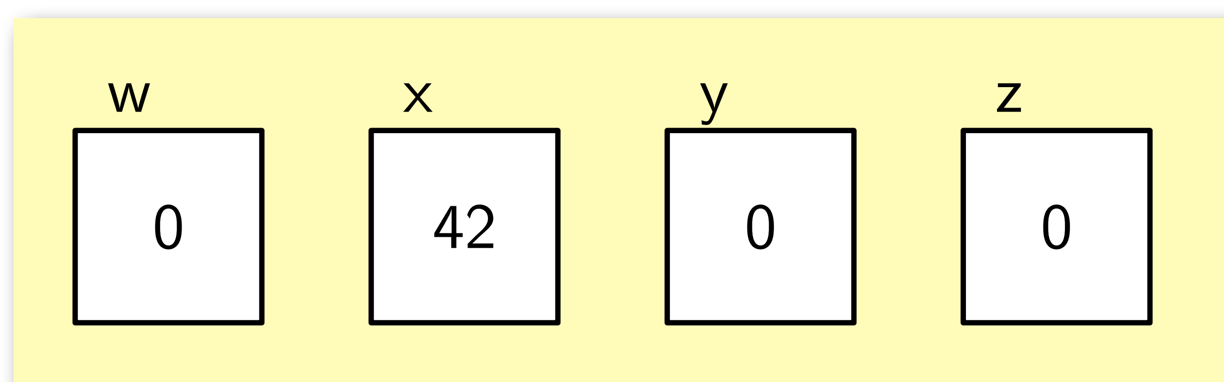

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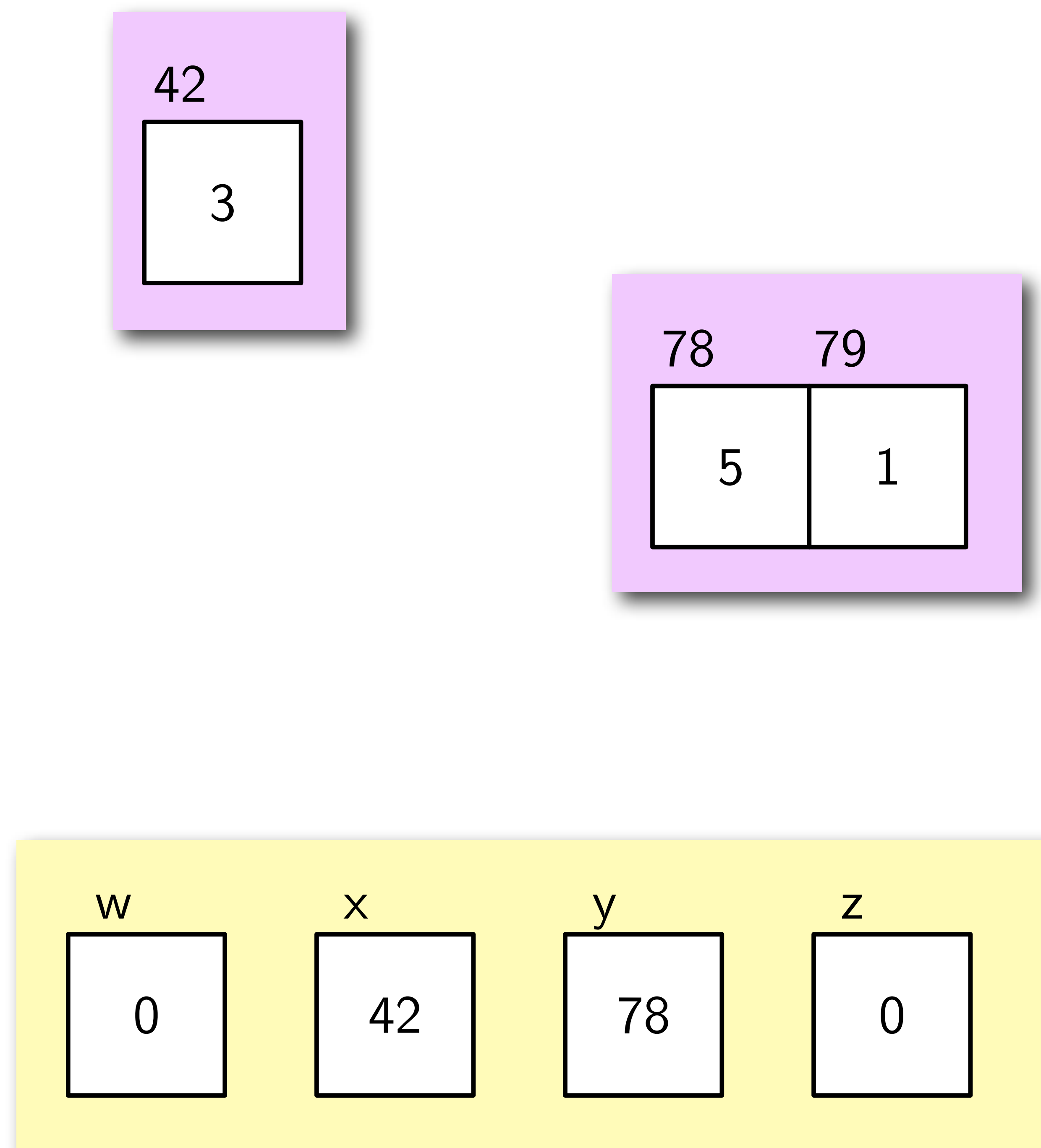

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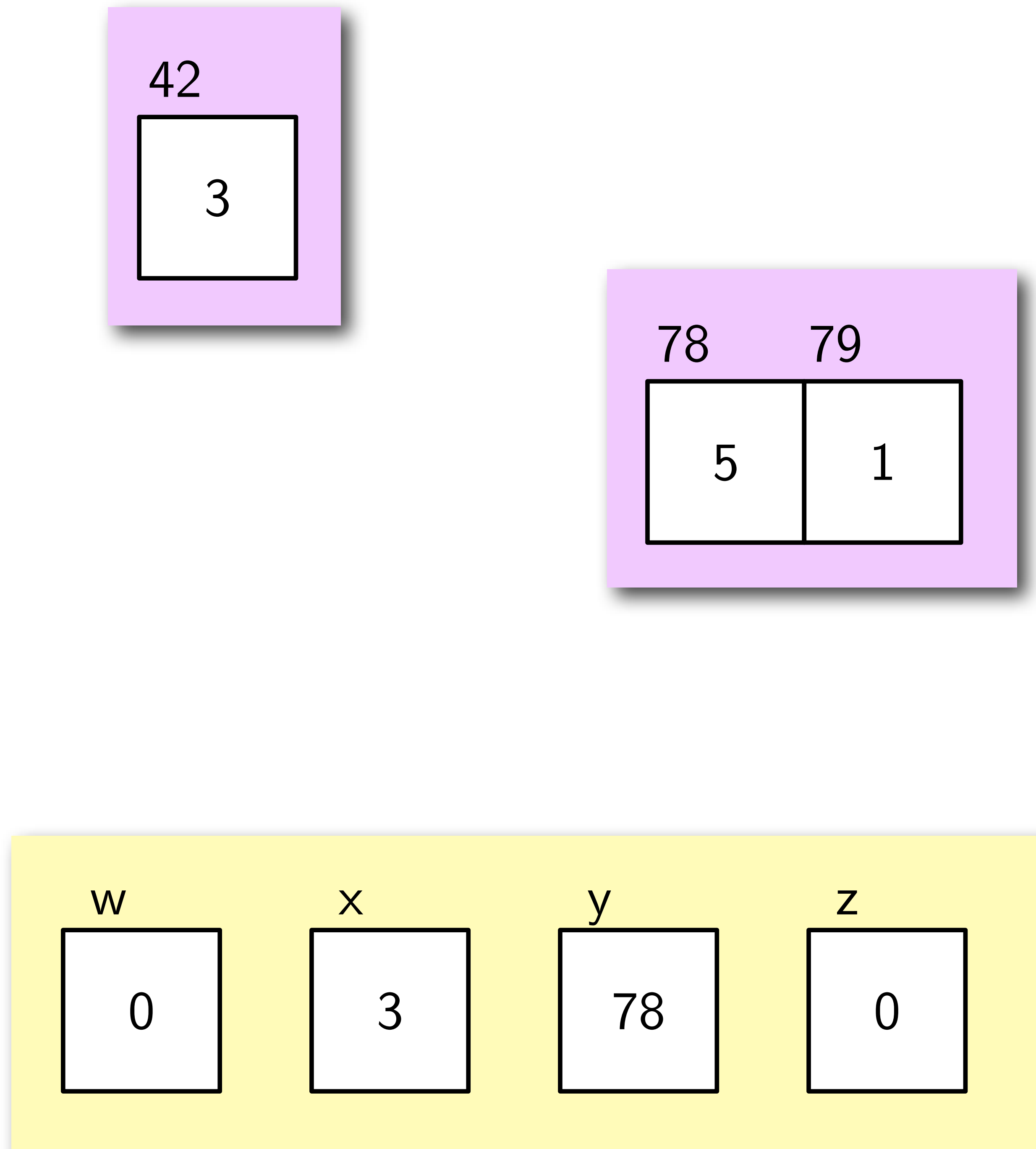

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  x := cons(3);
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  [y+1] := 6;
⇒ dispose(y);
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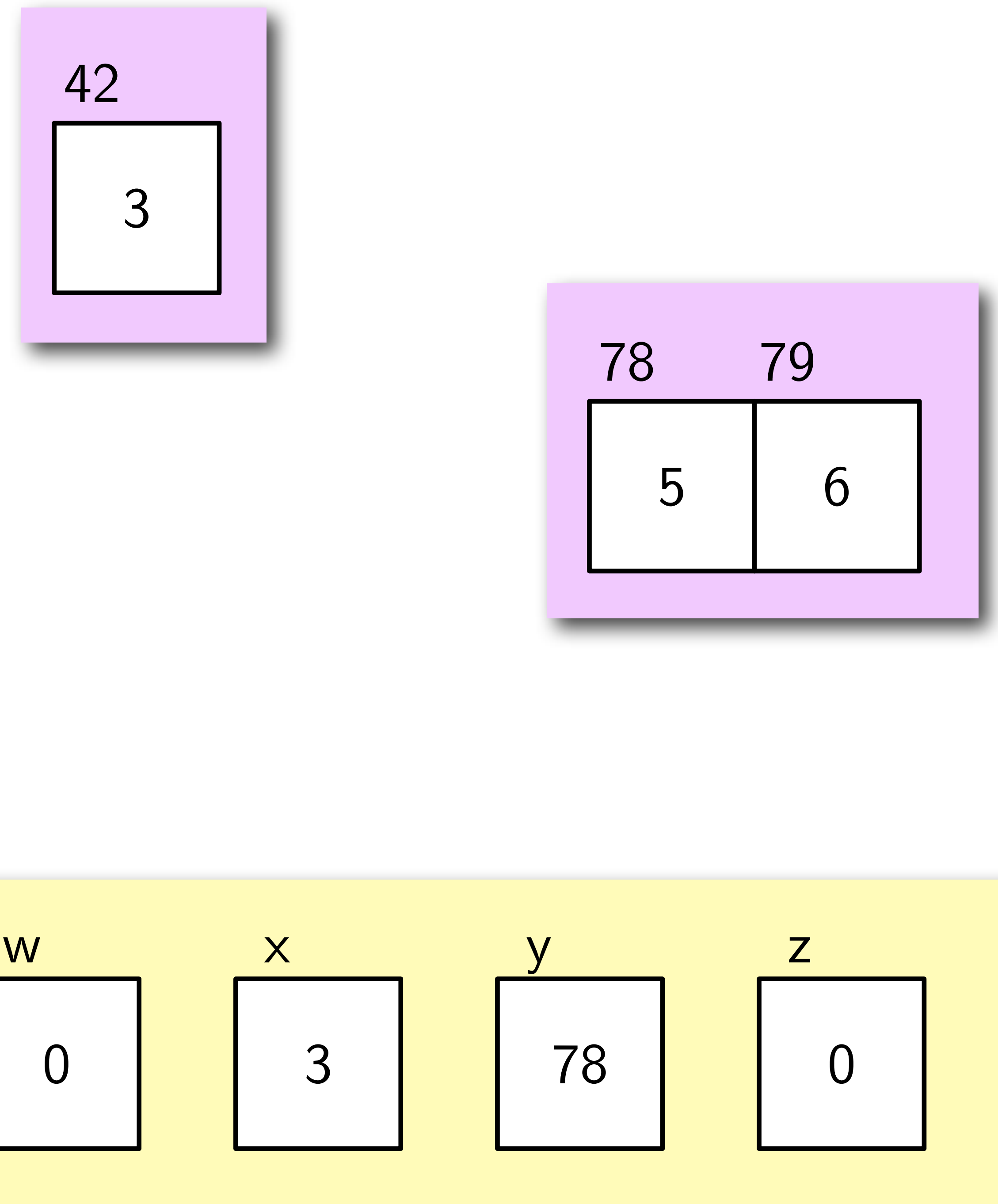

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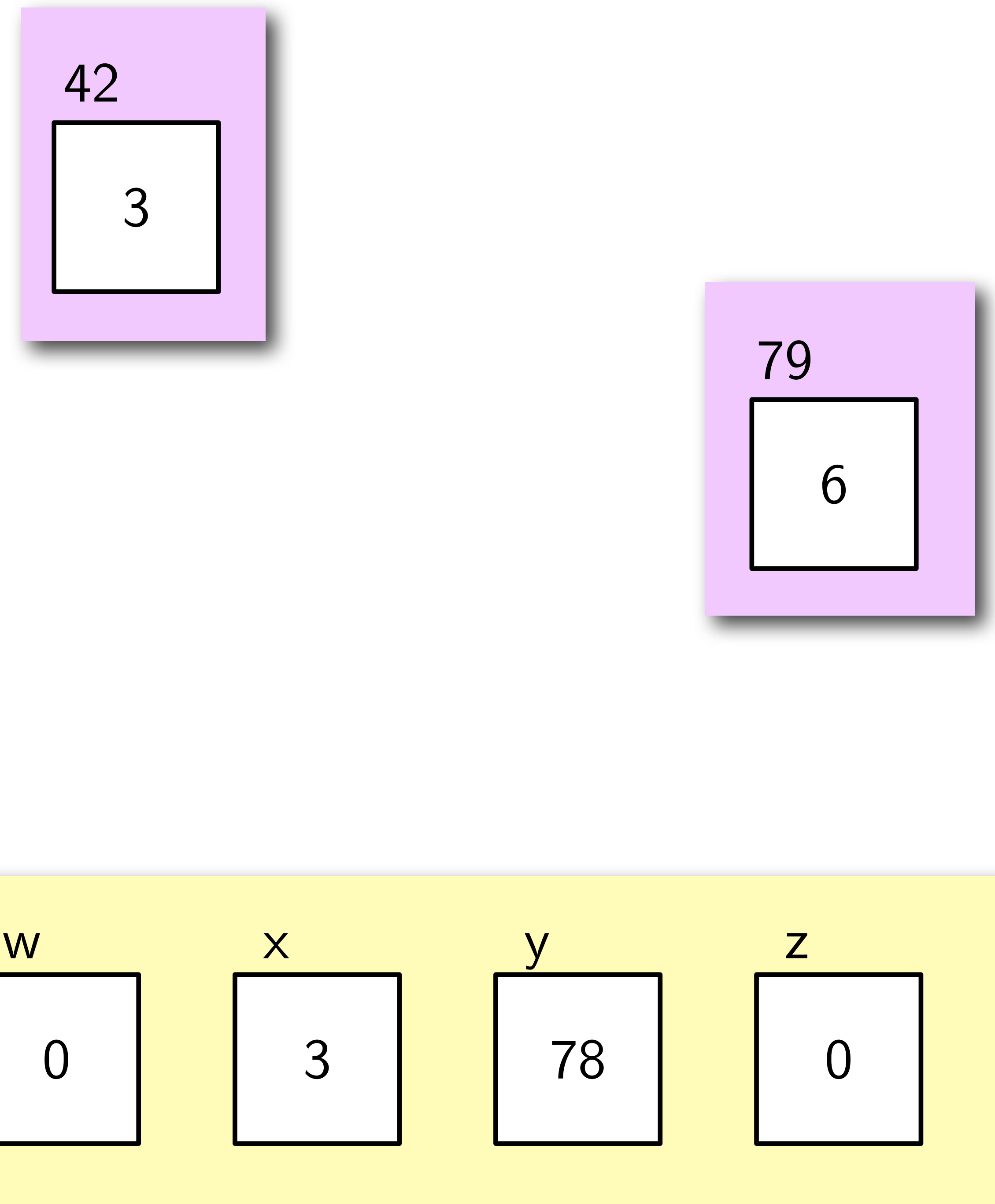

$\{\mathsf{emp}\}$

$x := \mathsf{cons}(3);$

$\{x \mapsto 3\}$

$y := \mathsf{cons}(5,1);$

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

$x := [x];$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

$[y+1] := 6;$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$

$\mathsf{dispose}(y);$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$\{\mathrm{emp}\}$
x := cons(3);
$\{x \mapsto 3\}$
y := cons(5,1);
$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$
x := [x];
$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$
[y+1] := 6;
$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$
dispose(y);
$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{\mathrm{emp}\}\ x := \mathrm{cons}(e_1,\ldots,e_n)\ \{x \mapsto e_1,\ldots,e_n\}}$$

$\{\mathsf{emp}\}$

$x := \mathsf{cons}(3);$

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$y := \mathsf{cons}(5,1);$

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

$x := [x];$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1$

$[y+1] := 6;$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1$

$\mathsf{dispose}(y);$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{\mathsf{emp}\}\ x := \mathsf{cons}(e_1,\ldots,e_n)\ \{x \mapsto e_1,\ldots,e_n\}}$$

$$\frac{\{P\}\ C\ \{Q\}}{\{P * R\}\ C\ \{Q * R\}}\ (\dagger)$$

†provided R doesn't mention any variable modified by C

$\{emp\}$

$x := cons(3);$

$\{x \mapsto 3 * emp\}$

$y := cons(5,1);$

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

$x := [x];$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto$

$[y+1] := 6;$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto$

$dispose(y);$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{emp\}\ x := cons(e_1,\ldots,e_n)\ \{x \mapsto e_1,\ldots,e_n\}}$$

$$\frac{\{P\}\ C\ \{Q\}}{\{P * R\}\ C\ \{Q * R\}}(\dagger)$$

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$\{\mathsf{emp}\}$

$\mathsf{x} := \mathsf{cons}(3);$

$\{\mathsf{x} \mapsto 3\}$

$\mathsf{y} := \mathsf{cons}(5,1);$

$\{\mathsf{x} \mapsto 3 * \mathsf{y} \mapsto 5 * \mathsf{y}+1 \mapsto 1\}$

$\mathsf{x} := [\mathsf{x}];$

$\{\exists \mathsf{X}.\ \mathsf{x} \doteq 3 * \mathsf{X} \mapsto 3 * \mathsf{y} \mapsto 5 * \mathsf{y}+1 \mapsto 1\}$

$[\mathsf{y}+1] := 6;$

$\{\exists \mathsf{X}.\ \mathsf{x} \doteq 3 * \mathsf{X} \mapsto 3 * \mathsf{y} \mapsto 5 * \mathsf{y}+1 \mapsto 6\}$

$\mathsf{dispose}(\mathsf{y});$

$\{\exists \mathsf{X}.\ \mathsf{x} \doteq 3 * \mathsf{X} \mapsto 3 * \mathsf{y}+1 \mapsto 6\}$

$\{\mathsf{emp}\}$

x := cons(3);

$\{x \mapsto 3\}$

y := cons(5,1);

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

x := [x];

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

[y+1] := 6;

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$

dispose(y);

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{e \mapsto Y\}\ x := [e]\ \{x \doteq Y * e \mapsto Y\}}$$

$\{\mathsf{emp}\}$

x := cons(3);

$\{x \mapsto 3\}$

y := cons(5,1);

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

x := [x];

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

[y+1] := 6;

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dispose(y);

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$$\frac{}{\{e \doteq X * X \mapsto Y\}\ x := [e]\ \{x \doteq Y * X \mapsto Y\}}$$

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x := cons(3);

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y := cons(5,1);

$\{\exists X.\ x \doteq X * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

x := [x];

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

[y+1] := 6;

$\{\exists X.\ x \doteq 3$ … $+1 \mapsto 6\}$

dispose(…

$\{\exists X.\ x \doteq$ …

$$\frac{}{\{e \doteq X * X \mapsto Y\}\ x := [e]\ \{x \doteq Y * X \mapsto Y\}}$$

$$\frac{\{P\}\ C\ \{Q\}}{\{\exists X.P\}\ C\ \{\exists X.Q\}}$$

$\{\mathsf{emp}\}$

$x := \mathsf{cons}(3);$

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$x := [x];$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

$[y+1] := 6;$

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$\mathsf{dispose}(y);$

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[y+1] := 6;

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$

dispose(y);

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{e_1 \mapsto \_\}\ [e_1] := e_2\ \{e_1 \mapsto e_2\}}$$

$\{emp\}$

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$\{x \mapsto 3\}$

y := cons(5,1);

$\{x \mapsto 3 * y \mapsto 5 * y+1 \mapsto 1\}$

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[y+1] := 6;

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dispose(y);

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y+1 \mapsto 6\}$

$$\frac{}{\{e \mapsto \_\}\ \text{dispose}(e)\ \{emp\}}$$

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x := cons(3);

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[y+1] := 6;

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$

dispose(y);

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * \mathsf{emp} * y+1 \mapsto 6\}$

$$\frac{}{\{e \mapsto \_\}\ \mathsf{dispose}(e)\ \{\mathsf{emp}\}}$$

$\{\mathsf{emp}\}$

$x := \mathsf{cons}(3);$

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$[y+1] := 6;$

$\{\exists X.\ x \doteq 3 * X \mapsto 3 * y \mapsto 5 * y+1 \mapsto 6\}$

$\mathsf{dispose}(y);$

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Soundness

$$\text{if} \quad \vdash \{P\}\ C\ \{Q\}$$
$$\text{then} \quad \models \{P\}\ C\ \{Q\}$$

$$\frac{\vdash \{P\}\ C\ \{Q\}}{\vdash \{P * R\}\ C\ \{Q * R\}}(\dagger)$$

†provided R doesn't mention any variable modified by C

$$\text{assume} \quad \models \{P\}\ C\ \{Q\}$$
$$\text{show} \quad \models \{P{*}R\}\ C\ \{Q{*}R\}$$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assume $\models$ {P} C {Q}

if $(P * R)\sigma$ then:

(C,σ) doesn't fault, and $(C,\sigma) \Downarrow \sigma' \Rightarrow (Q * R)\sigma'$

show $\models$ {P∗R} C {Q∗R}

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

(C,σ) doesn't fault, and $(C,\sigma)\Downarrow\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P * R)\sigma$

(C,σ) doesn't fault

$(C,\sigma) \Downarrow \sigma' \Rightarrow (Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \land R\ \sigma_2$

(C,σ) doesn't fault

$(C,\sigma)\Downarrow\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P * R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge \ R\ \sigma_2$

(C, σ_1) doesn't fault

(C, σ) doesn't fault

$(C, \sigma) \Downarrow \sigma' \Rightarrow (Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \wedge R\ \sigma_2$

(C,σ_1) doesn't fault

Safety Monotonicity

(C,σ) doesn't fault

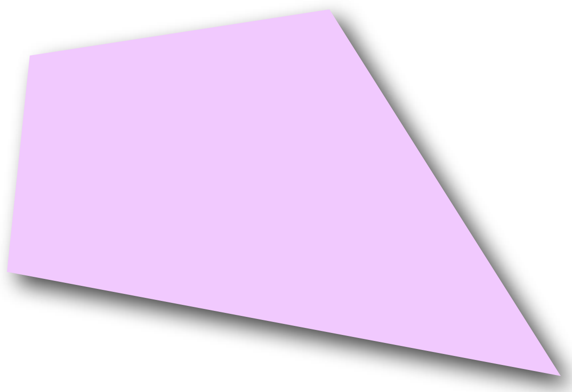
$(C,\sigma){\Downarrow}\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assum

$(P*$



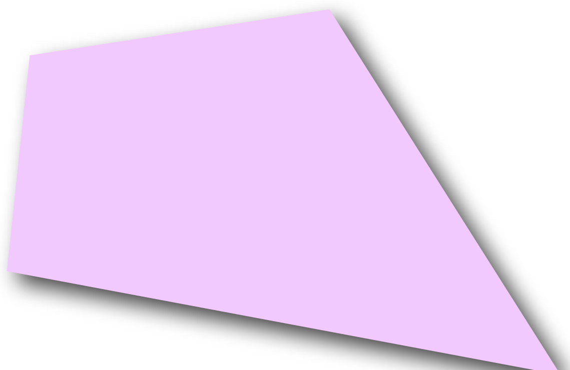
Safety Monotonicity

(C,σ) doesn't fault

$(C,\sigma)\Downarrow\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule



C doesn't fault

Safety Monotonicity

(C,σ) doesn't fault

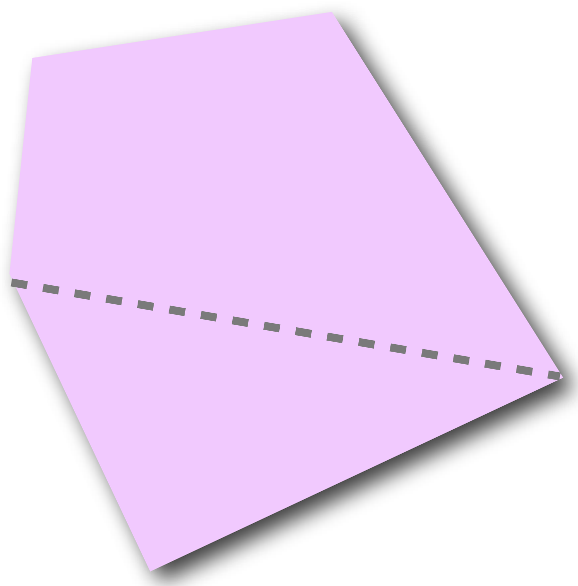
$(C,\sigma)\Downarrow\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assum

(P∗



Safety Monotonicity

(C,σ) doesn't fault

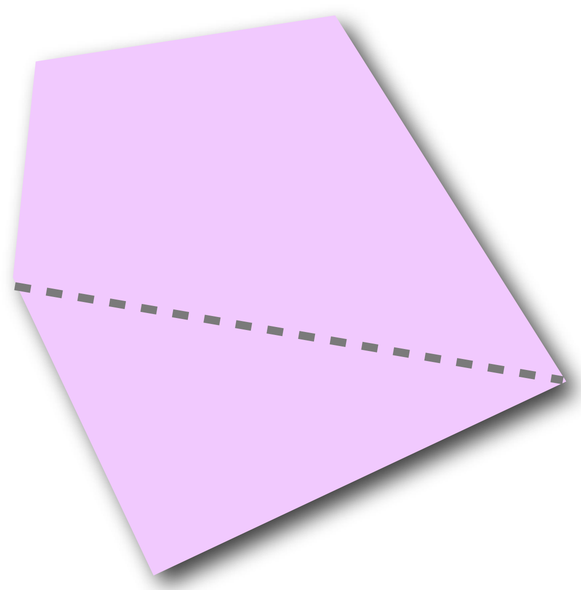
(C,σ)⇓σ' ⇒ (Q∗R)σ'

show ⊧ {P∗R} C {Q∗R}

Soundness of Frame rule

assum

(P∗



C still doesn't fault

Safety Monotonicity

(C,σ) doesn't fault

(C,σ)⇓σ' ⇒ (Q∗R)σ'

show ⊨ {P∗R} C {Q∗R}

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P{*}R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \wedge R\ \sigma_2$

(C,σ_1) doesn't fault

Safety Monotonicity

(C,σ) doesn't fault

$(C,\sigma)\Downarrow\sigma' \Rightarrow (Q{*}R)\sigma'$

show $\models \{P{*}R\}\ C\ \{Q{*}R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge\ R\ \sigma_2$

(C,σ_1) doesn't fault

$(C,\sigma)\Downarrow\sigma' \Rightarrow (Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge\ \ R\ \sigma_2$

$(C,\sigma)\Downarrow\sigma'$

(C,σ_1) doesn't fault

$(Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P * R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge \ R\ \sigma_2$

$(C,\sigma) \Downarrow \sigma'$

(C,σ_1) doesn't fault

Frame Property

$\sigma' = \sigma_1' * \sigma_2$

$(C,\sigma_1) \Downarrow \sigma_1'$

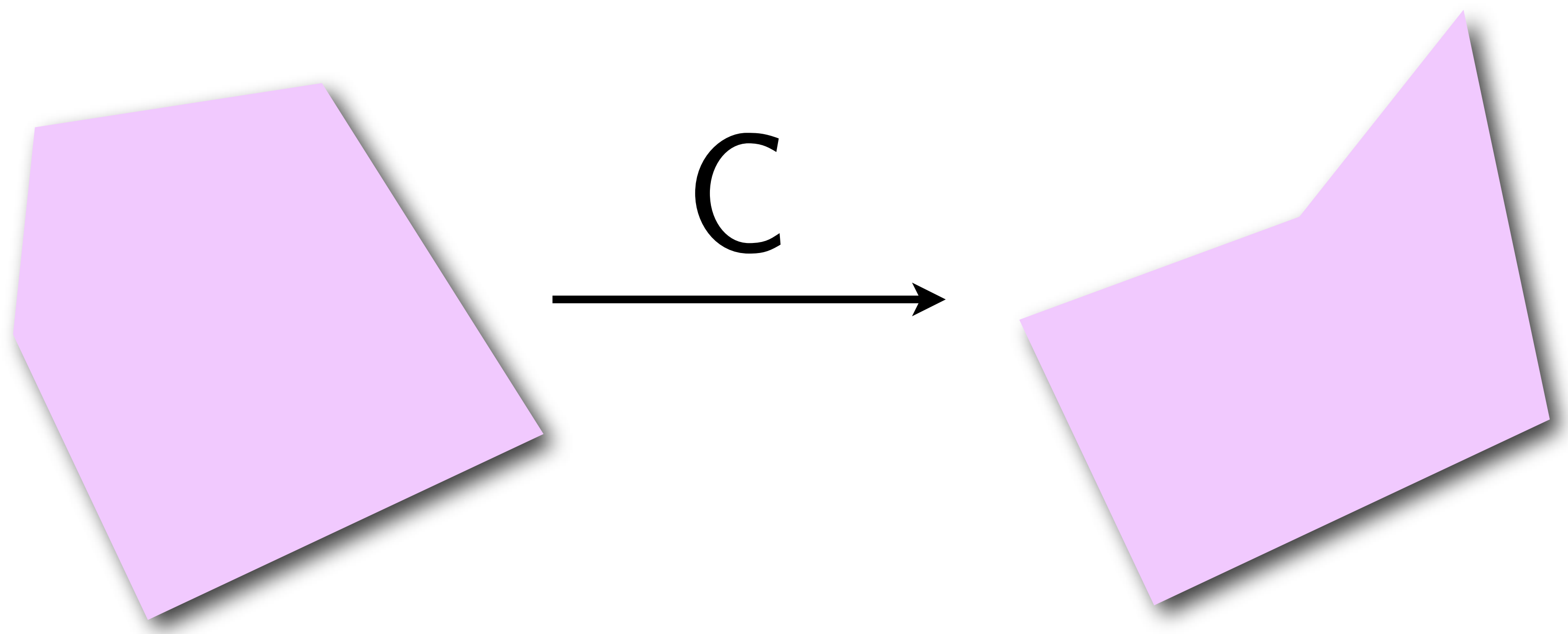
$(Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assum

$(P*$



Frame Property

$(C,\sigma_1) \Downarrow \sigma_1$

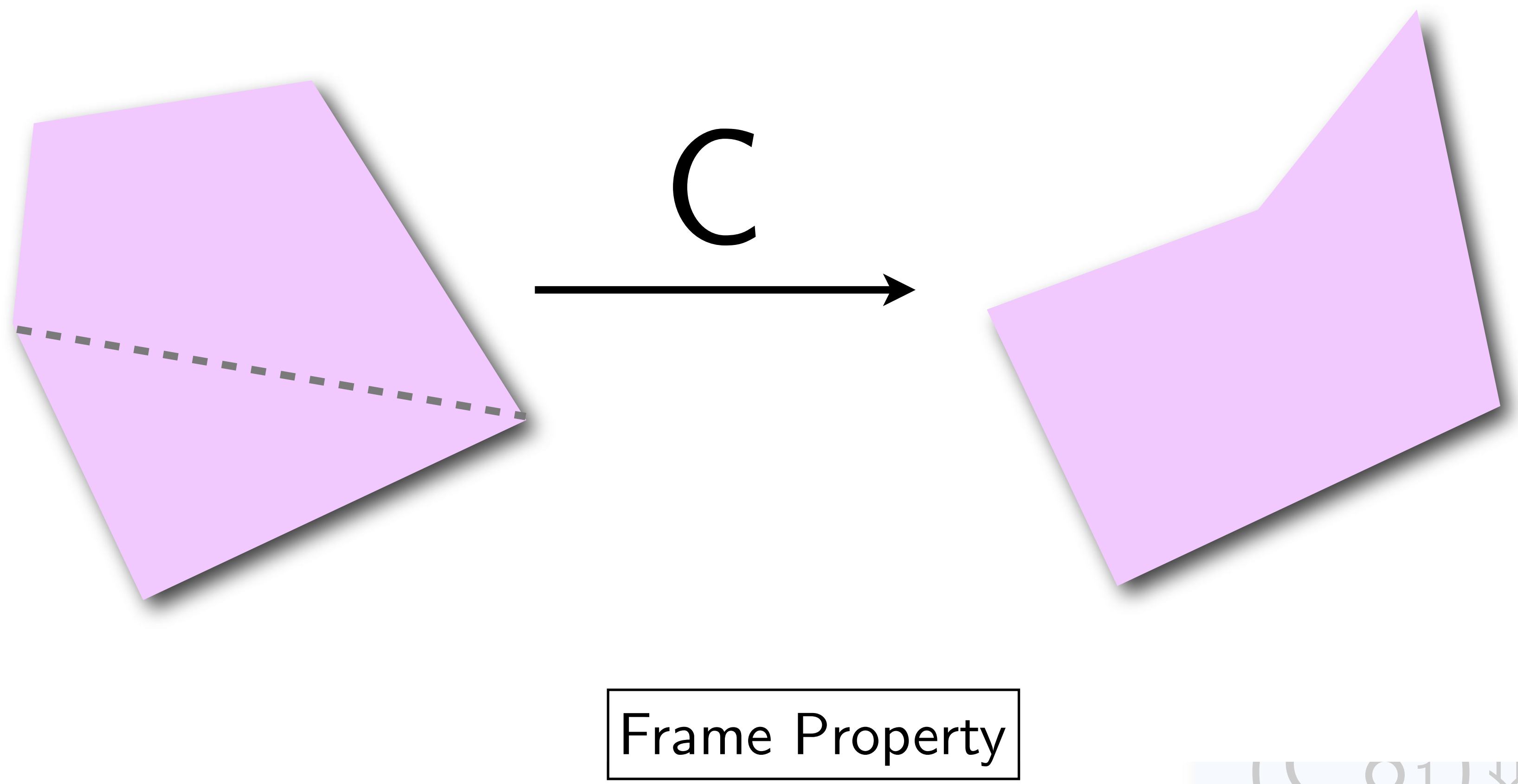
$(Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assum

(P∗



$(C,\sigma_1) \Downarrow \sigma_1$

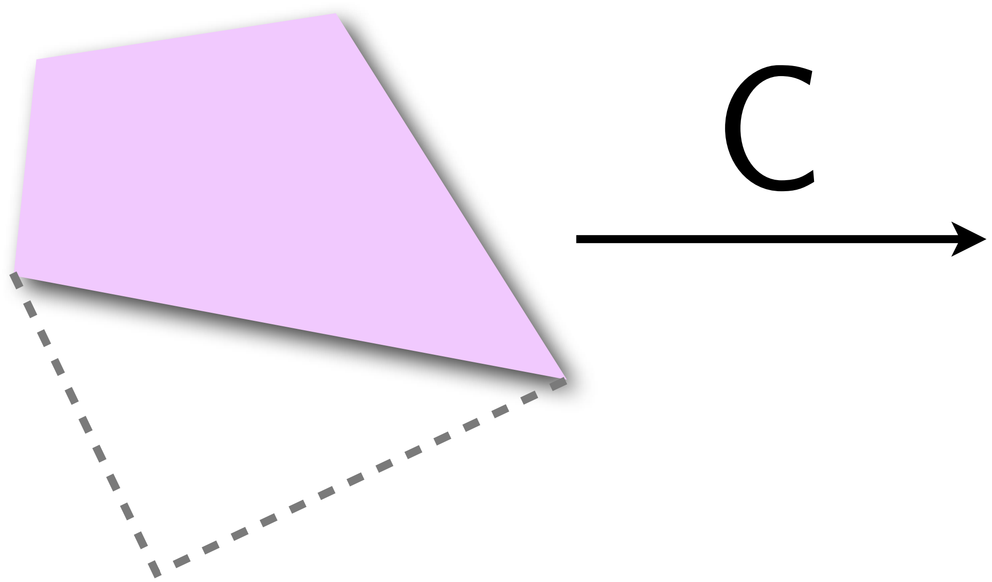
$(Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assum

(P∗



Frame Property

$(C,\sigma_1)\Downarrow\sigma_1$

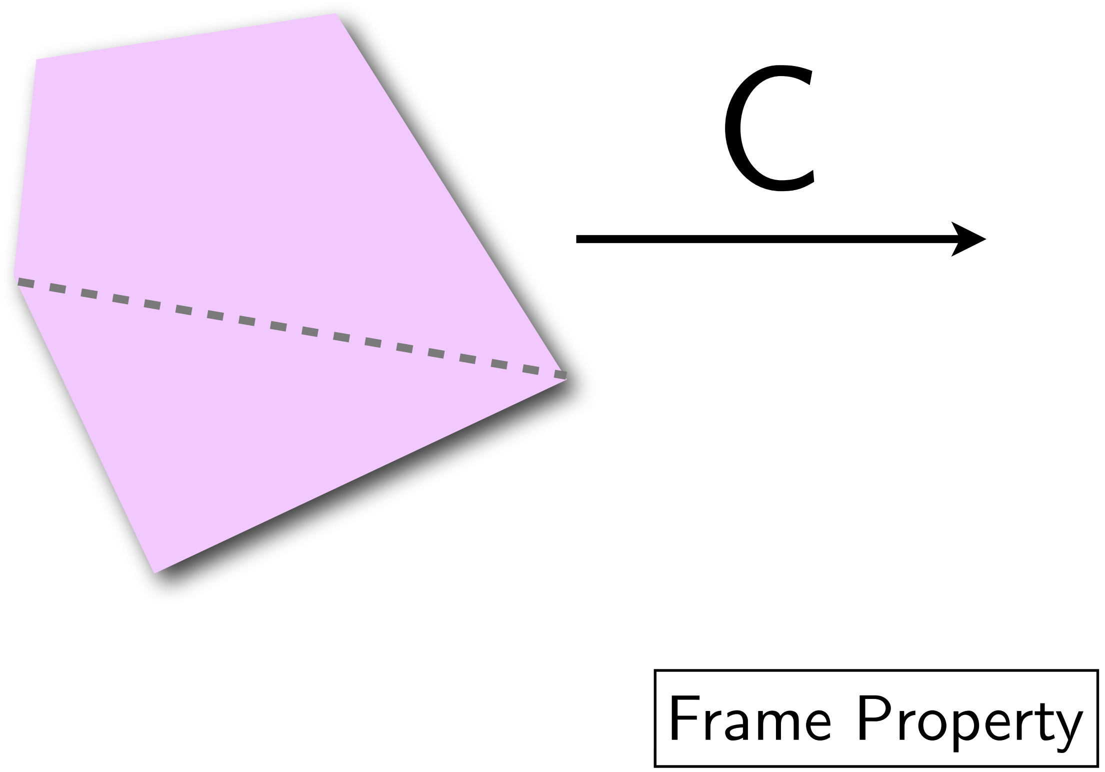
$(Q*R)\sigma'$

show $\models$ {P∗R} C {Q∗R}

Soundness of Frame rule

assum

(P∗



$(C,\sigma_1) \Downarrow \sigma_1$

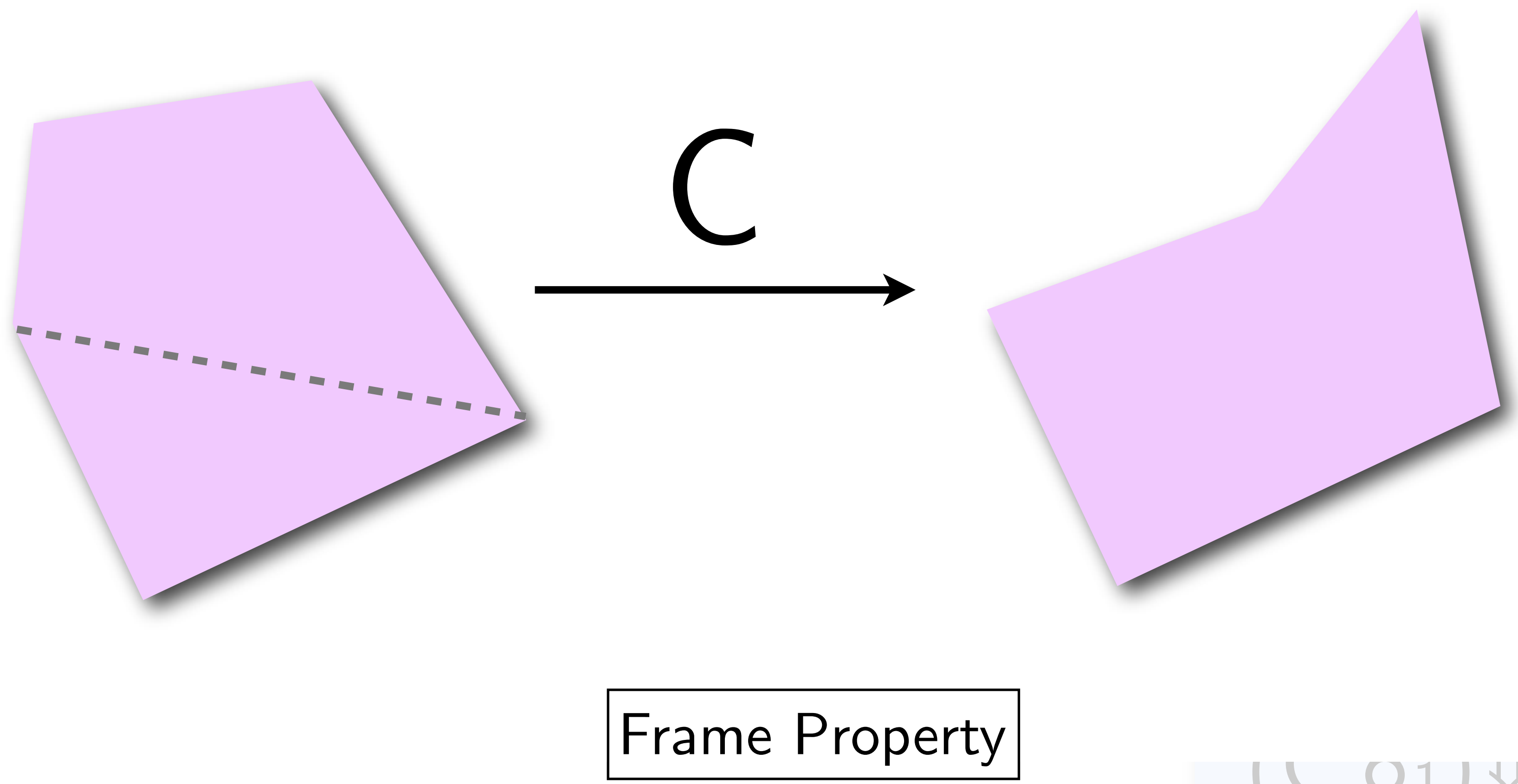
$(Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assum

(P∗



$(C,\sigma_1) \Downarrow \sigma_1$

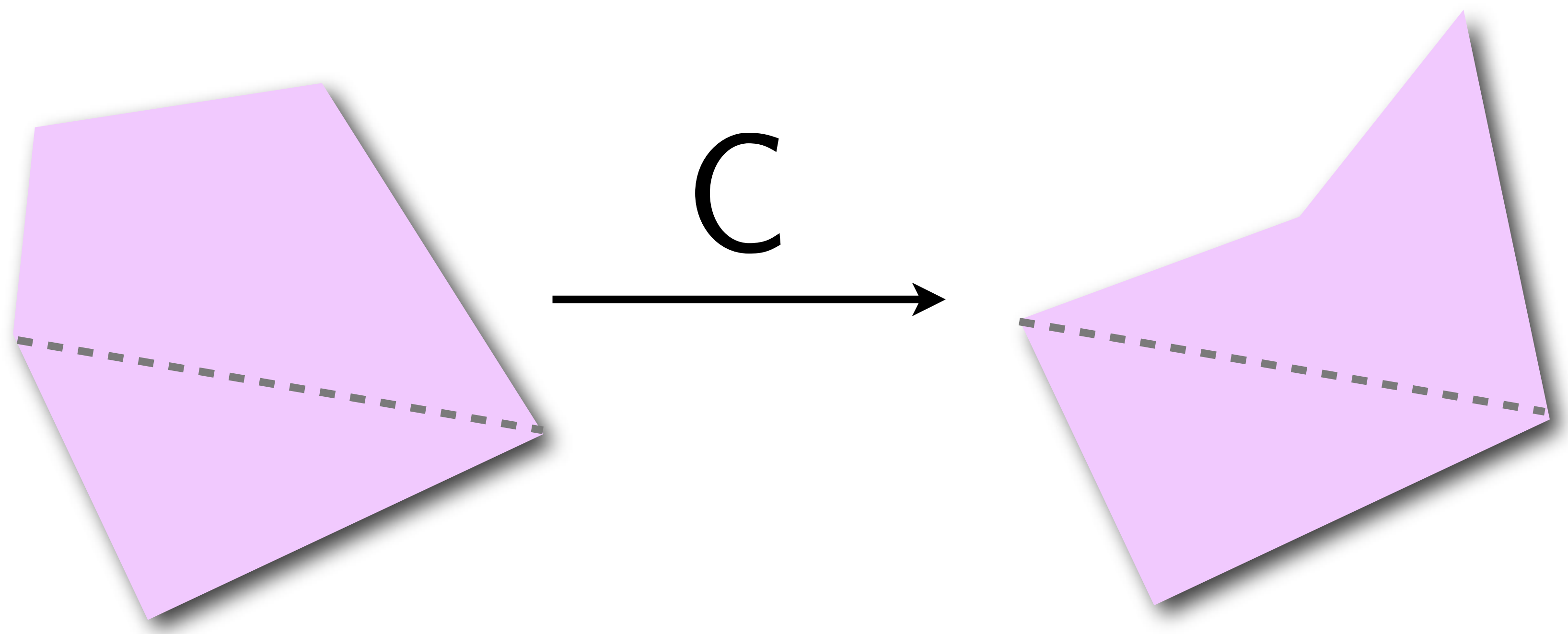
$(Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assum

(P∗



Frame Property

(C,σ1)⇓σ1

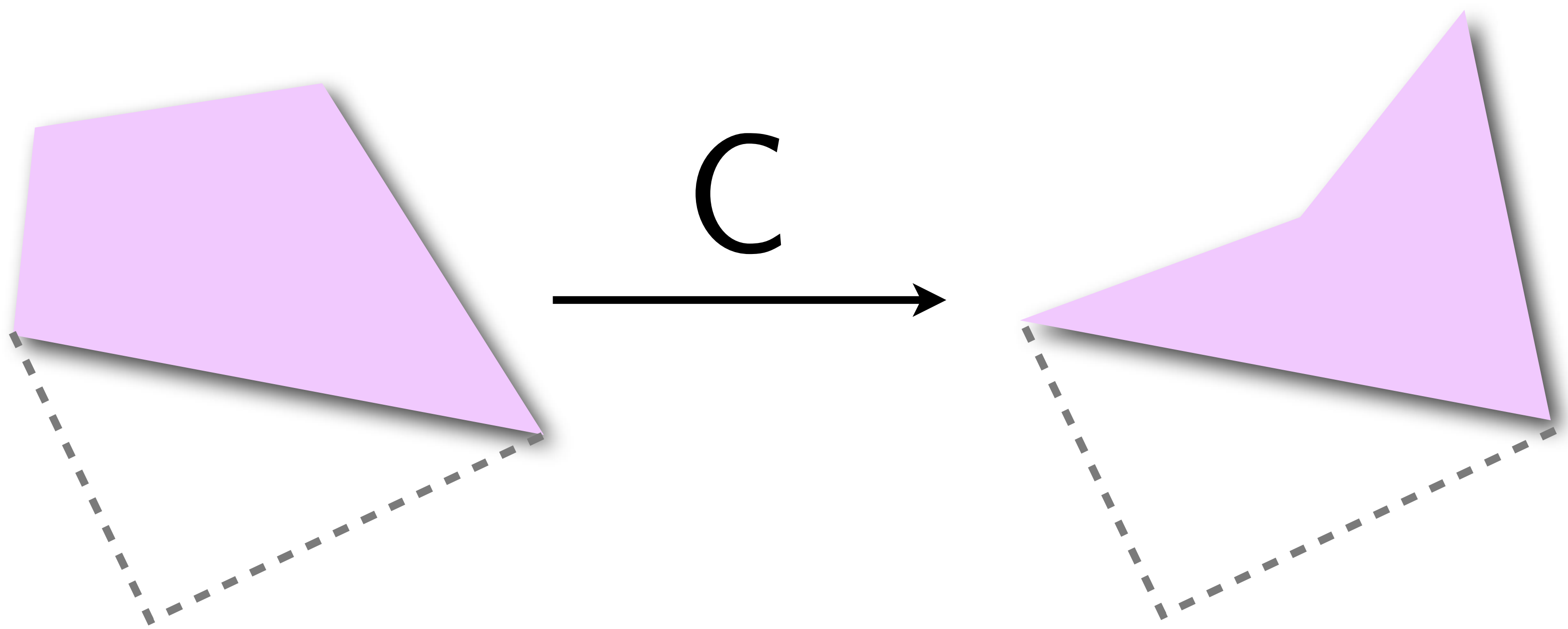
(Q∗R)σ'

show ⊨ {P∗R} C {Q∗R}

Soundness of Frame rule

assum…

(P∗…



Frame Property

$(C,\sigma_1) \Downarrow \sigma_1$

$(Q * R)\sigma'$

show $\models \{P * R\}\ C\ \{Q * R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \wedge R\ \sigma_2$

$(C,\sigma) \Downarrow \sigma'$

(C,σ_1) doesn't fault

Frame Property

$\sigma' = \sigma_1' * \sigma_2$

$(C,\sigma_1) \Downarrow \sigma_1'$

$(Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P{*}R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge\ \ R\ \sigma_2$

$\sigma' = \sigma_1' * \sigma_2$

$(C,\sigma_1) \Downarrow \sigma_1'$

$(Q{*}R)\sigma'$

show $\models \{P{*}R\}\ C\ \{Q{*}R\}$

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

$(P*R)\sigma$

$\sigma = \sigma_1 * \sigma_2$

$P\ \sigma_1 \ \wedge \ R\ \sigma_2$

$\sigma' = \sigma_1' * \sigma_2$

$Q\ \sigma_1'$

$(C,\sigma_1) \Downarrow \sigma_1'$

$(Q*R)\sigma'$

show $\models \{P*R\}\ C\ \{Q*R\}$

Soundness of Frame rule

assume $\models$ {P} C {Q}

show $\models$ {P∗R} C {Q∗R}

Soundness of Frame rule

assume $\models \{P\}\ C\ \{Q\}$

show $\models \{P * R\}\ C\ \{Q * R\}$

Summary

- A old-style proof of `list_reverse`
- A proof of `list_reverse` in separation logic
- Separation logic's proof rules
- Soundness of the Frame rule

$$(P * Q)\ s = \exists s_1, s_2.\ s = s_1 + s_2 \text{ and } (P\ s_1) \text{ and } (Q\ s_2)$$

$$(P \wedge Q)\ s = (P\ s) \text{ and } (Q\ s)$$

$$(P \vee Q)\ s = (P\ s) \text{ or } (Q\ s)$$

$$(\neg P)\ s = \text{not } (P\ s)$$

 $s = s \geq £5$

 $s = s \geq £20$

( $\wedge$ ) s = ( s) and ( s)

$$= s \geq £5 \text{ and } s \geq £20$$

$$= s \geq £20$$

$(P * Q)\ s \ = \ \exists s_1, s_2.\ s = s_1 + s_2 \text{ and } (P\ s_1) \text{ and } (Q\ s_2)$

$(P \wedge Q)\ s \ = \ (P\ s) \text{ and } (Q\ s)$

$(P \vee Q)\ s \ = \ (P\ s) \text{ or } (Q\ s)$

$(\neg P)\ s \ = \ \text{not } (P\ s)$

 $s \ = \ s \geq £5$

 $s \ = \ s \geq £20$

( $*$ ) $s \ = \ \exists s_1, s_2.\ s = s_1 + s_2 \text{ and }$ ( s_1) and ( s_2)

$= \ \exists s_1, s_2.\ s = s_1 + s_2 \text{ and } s_1 \geq £5 \text{ and } s_2 \geq £20$

$= \ s \geq £25$